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Noise-Induced Excitability of the Complex Liquid Flows

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Abstract

We study an excitability for the stochastically forced system modeling a dynamics of the complex liquid flows. A phenomenon of noise-induced generation of large-amplitude oscillations in a zone of stable equilibria is studied.

Keywords: Excitability, stochastic disturbances, complex liquid flows

Deterministic model

Consider a complex liquid flow bounded by two parallel planes $z = 0,\ z = h$. The lower plane $z = 0$ is fixed and the unidirectional shear stress Σ is applied to the upper plane $z = h$. Due to the symmetry, one can limit by one space variable z. The physical state of the system is uniquely determined by the function of the viscous stress $\sigma(t, z)$. This function in $[0, \infty) \times [0, h]$ satisfies the equation [1]

$$\rho\frac{\partial}{\partial t}\left(f(\sigma) + G\frac{\partial \sigma}{\partial t}\right) = \frac{\partial^2 \sigma}{\partial z^2} \tag{1}$$

with boundary conditions

$$\sigma(t, h) = \Sigma, \quad \frac{\partial \sigma}{\partial z}(t, 0) = 0. \tag{2}$$

Here, the function $f(\sigma)$ reflects the nonlinear N-shaped character of the shear rate as a function of the stress (see Fig.1), ρ is a medium density, G is a relaxation parameter.

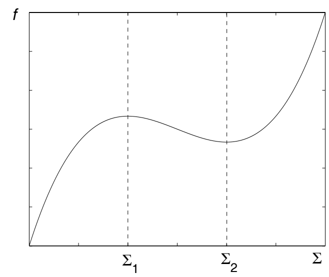


Fig.1. Plot of the function $f(\Sigma)$.

In this paper, for the distributed model of the flow stream (1)-(2) a three-layer discretization is used. For lines $z = z_i$ ($z_0 = 0, z_1 = \frac{h}{2}, z_2 = h$) consider corresponding approximations $\sigma_i(t)$ for functions $\sigma(t, z_i)$. Using formulas of numerical differentiation, one get the following approximation of the equation (1) on the line $z = z_1$

$$\rho\left(\frac{df(\sigma_1)}{d\sigma}\frac{d\sigma_1}{dt} + G\frac{d^2\sigma_1}{dt^2}\right) = 4\frac{\sigma_0(t) - 2\sigma_1(t) + \sigma_2(t)}{h^2}. \tag{3}$$

It follows from boundary conditions (2) that

$$\sigma_2(t) = \Sigma,\ \sigma_1(t) - \sigma_0(t) = 0. \tag{4}$$

Equations (3),(4) imply

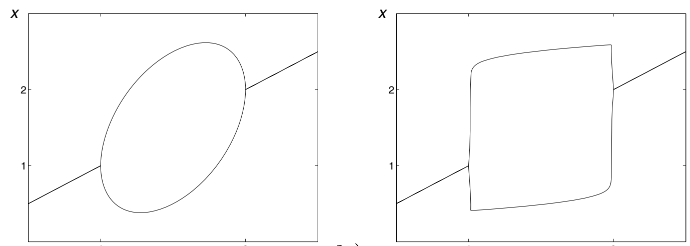
$$G\rho\frac{d^2\sigma_1}{dt} + \rho f'(\sigma_1)\frac{d\sigma_1}{dt} = 4\frac{\Sigma - \sigma_1}{h^2}. \tag{5}$$

A solution $\sigma_1(t) \equiv \Sigma$ of the equation (5) is a unique equilibrium. The equation (5) for the variables $x = \sigma_1,\ \ y = \frac{d\sigma_1}{dt}$ can be rewritten as a system

$$\dot{x} = y, \qquad \dot{y} = -\frac{4}{G\rho h^2}x - \frac{f'(x)}{G}y + \frac{4}{G\rho h^2}\Sigma. \tag{6}$$

For a study of the possible dynamical regimes of this system we fix parameters $G = \rho = h = 1,\ f(x) = k(x^3/3 - x^2 + x/2)$. Here, $f'(x) = k(x^2 - 3x + 2)$. The function $f(x)$ models a characteristic type of N-shaped curve mentioned above, and the parameter k reflects a stiffness of this nonlinearity.

The equilibrium $x = \Sigma, y = 0$ of the system (6) is stable for $0 < \Sigma < 1$ and $\Sigma > 2$. On the interval $1 < \Sigma < 2$, this equilibrium is unstable and a stable limit cycle is observed. The extreme values of the variable x for attractors of the system (equilibria and cycles) on the interval $0 < \Sigma < 3$ for $k = 1,\ 20$ are plotted. The instability of the equilibrium for $\Sigma \in (1, 2)$ leads to the appearance of the large-amplitude auto-oscillations of the flow.



a) b)

Fig.2. Extreme values x for the attractors: a) $k = 1$, b) $k = 20$.

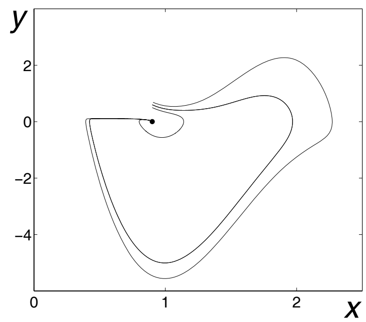


Fig.3. Phase portrait of the deterministic system (6) for $\Sigma = 0.9$ and $k = 20$.

In this paper, we focus on the zone $\Sigma < 1$ close to the point $\Sigma_1 = 1$ of Andronov-Hopf bifurcation. In Fig.3, a phase portrait of the deterministic system (6) for $\Sigma = 0.9$ is plotted. Small deviations of the initial state from the equilibrium result in small-amplitude trajectories that correspond to *subthreshold* responses. If we take initial deviations larger than some threshold, large-amplitude trajectories appear that correspond to *suprathreshold* response. Around the equilibrium, one can find a set of initial points corresponding to the subthreshold response. A size of this subthreshold domain essentially depends on the parameter Σ. The closer Σ to the bifurcation value, the less a size of this subthreshold domain.

Such non-uniformity of the phase portrait is a underlying reason of the stochastic excitability of the studied system. Noise-induced excitability was studied for various dynamical systems [2-5].

Stochastic model

For the analysis of stochastic effects, consider the randomly forced system

$$\dot{x} = y, \qquad \dot{y} = -4x - k(x^2 - 3x + 2)y + 4\Sigma + \varepsilon\dot{w}(t), \tag{7}$$

where $w(t)$ is a standard Wiener process, ε is a noise intensity.

In Figs.4-5, random trajectories and time series of the system (7) for two values of the noise intensity are plotted. For weak noise $\varepsilon = 0.1$, random trajectories leave the stable equilibrium and concentrate in the subthreshold zone.

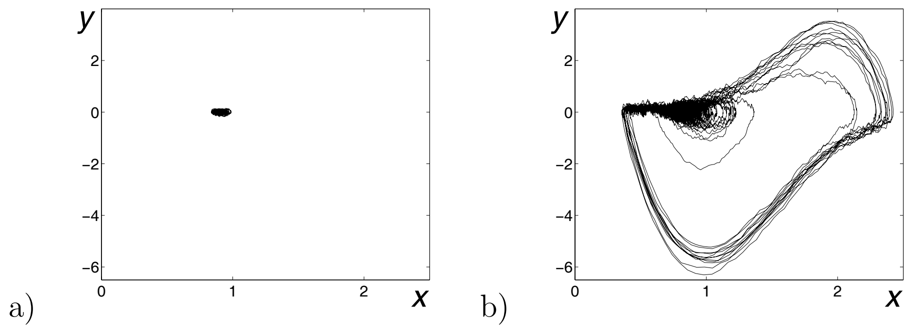


Fig.4. Random trajectories of the system (7): a) for $\varepsilon = 0.1$, b) for $\varepsilon = 0.5$.

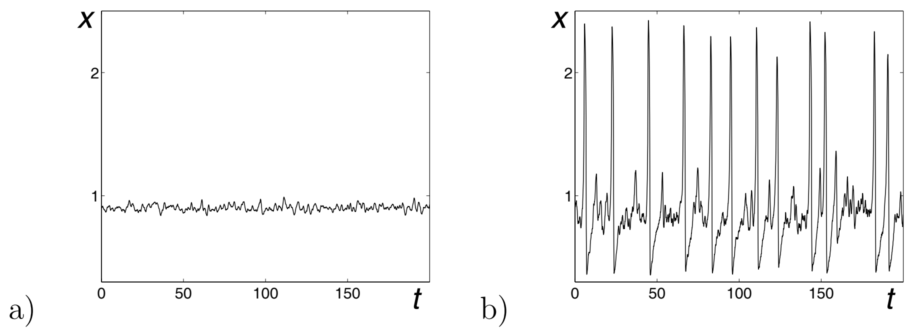


Fig.5. Random time series: a) for $\varepsilon = 0.1$, b) for $\varepsilon = 0.5$.

Here, time series demonstrate small-amplitude stochastic oscillations near the equilibrium (see Fig.5a). As the noise intensity increases, random trajectories transit to the suprathreshold zone. As one can see in Fig.4b for $\varepsilon = 0.5$, the system exhibits stochastic oscillations of large amplitude. An intermittency of small- and large-amplitude oscillations is clearly seen in Fig.5b. So, this system is highly excitable to stochastic disturbances. This model exhibits a noise-induced stochastic cycle even when the deterministic system has a stable equilibrium only.

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